姜秉介 教授指導
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On Fixed Point Theorems for Fuzzy Monotone Maps

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誠信女子大學校 教育大學院
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認 准 書

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논 문 개 요

일반순서집합(一般順序集合)에서의 극대원리(極大原理)들이 부동점정리(不動點定理)로 변환될 수 있음은 잘 알려진 사실이다.

Beg [2]은 퍼지순서집합(fuzzy ordered set)에서 퍼지 Zorn의 보조정리(fuzzy Zorn's lemma)가 성립함을 보였다. 또 그와 관련한 단가(單價) 및 다가(多價)의 퍼지 함수에 대한 부동점정리들이 Beg [1, 2, 3] 와 Stouti [10] 에 의해 연구되었다.

우리는 이 논문에서 퍼지 Zorn의 보조정리가 일반 순서집합에서의 보통의 Zorn의 보조정리로부터 증명될 수 있음을 보인다. 이것을 증명하기 위해서 먼저 Hausdorff 극대원리(Hausdorff maximality principle)가 퍼지 순서집합에서도 성립함을 보인다.

퍼지순서집합에서 단가 및 다가의 퍼지 함수에 대한 몇 가지 부동점정리들을 증명한다. 또 순서가 주어진 Banach 공간에서의 다가연산자(多價演算子)에 대한 Huy 와 Khanh의 [6] 부동점정리의 퍼지집합에서의 경우를 얻는다. 이것은 단가의 퍼지 함수에 대한 Beg[2] 의 부동점정리의 일반화이다.

목　　차

I. Introduction

Let X be a nonempty set. A reflexive and transitive relation $\leq$ on X is called a *quasi-order* on X. A *partial order* is an antisymmetric quasi-order. If X is endowed with a quasi-order(partial order, resp.) $\leq$, then $(X, \leq)$ is called a *quasi-ordered set*(*partially ordered set*, resp.).

The origin of fixed point theorems for functions on quasi-ordered sets is the following (See Dunford and Schwartz [4]);

Theorem (Zermelo). *Let X be a partially ordered set and assume that every chain in X has a supremum. Then every selfmap $f : X \to X$ satisfying*

$$x \leq f(x), \quad x \in X$$

has a fixed point.

The above fixed point theorem is equivalent to the following Zorn's Lemma.

Theorem (Zorn's Lemma). *Let $(X, \leq)$ be a quasi-ordered set, i.e $\leq$ is a reflexive and transitive relation on X. If every totally ordered subset of X has an upper bound, then X has a maximal element.*

Park [8] showed that Zorn's lemma can be reformulated to some fixed point theorems on ordered sets. Also he obtained some fixed point theorems for multivalued functions on ordered spaces [9].

Tarski [11] showed that every increasing selfmap of a complete lattice has a fixed point. And Huy [5] and Huy and Khanh [6] proved some fixed point theorems for multivalued increasing functions on ordered sets.

Recently, Beg [1] proved fuzzy Zorn's Lemma for fuzzy ordered sets. And some fixed point theorems for fuzzy monotone maps have been appeared in Beg [2,3] and Stouti [10]. In some sense their theorems are fuzzy version of those of Huy [5] and Huy and Khanh [6].

In this paper, we will show that fuzzy Zorn's lemma and fuzzy fixed point theorems for fuzzy monotone functions can be derived from usual Zorn's lemma and general fixed point theorems on ordered sets.

Section II deals with some basic concepts and preliminary results. In Section III, we will show that fuzzy Zorn's lemma can be derived from usual Zorn's lemma. To do this, we show that Hausdorff maximality principle holds in fuzzy ordered sets.

We will prove some fixed point theorems for single valued and multivalued fuzzy maps. We obtain a fuzzy version of Huy and Khanh's fixed point theorem for multivalued operators in ordered Banach space[6]. It is a generalization of the fixed point theorem of Beg [2] for single valued fuzzy monotone map.

II. Preliminaries

Let X be a nonempty set. A reflexive and transitive relation $\leq$ on X is called a *quasi-order* on X. If X is endowed with a quasi-order $\leq$, then $(X, \leq)$ is called a *quasi-ordered set.*

Let $(X, \leq)$ be a quasi-ordered set. A nonempty subset $A \subseteq X$ is said to be *totally ordered* if for any $a, b \in A$, $a \leq b$ or $b \leq a$. A totally ordered subset of X is called a *chain.*

For $A \subseteq X$, an element $u \in X$ is called an *upper bound* of A if $a \leq u$ for all $a \in A$. Also an element $l \in X$ is called a *lower bound* of A if $l \leq a$ for all $a \in A$. Also u is called a *supremum* of A and is denoted by $u = \sup A$ if u is an upper bound of A and for all upper bound x of A, $u \leq x$ holds. Similarly an element $l \in X$ is called an *infimum* of A and is denoted by $l = \inf A$ if l is a lower bound of A and for all lower bound x of A, $x \leq l$ holds.

Let $(X, \leq)$ be a quasi-ordered set. An element $a \in X$ is called *maximal* (*minimal*, resp.) element if

$$a \leq x \Rightarrow x \leq a \quad (x \leq a \Rightarrow a \leq x, \text{resp.})$$

for all $x \in X$. Obviously, if $\leq$ is a partial order and $a \in X$ is a maximal (minimal, resp.) element, then for all $x \in X$, $a \leq x (x \leq a$ resp.) implies that $x = a$.

Now consider the following definitions for fuzzy sets and fuzzy relations.

Definition II. 1. Let X be a nonempty set. A *fuzzy subset* A of X is characterized by its membership function

$$\mu_A : X \to [0, 1].$$

For any $x \in X$, $\mu_A(x)$ is interpreted as *the degree of membership* of x in A.

For any $x \in X$, we denote $\{x\}$ to be the fuzzy subset characterized by

$$\mu_{\{x\}}(y) = \begin{cases} 1, & \text{if } x = y \\ 0, & \text{otherwise.} \end{cases}$$

A fuzzy subset A is said to be *empty* if μ_A is the constant function 0.

Let A, B be two fuzzy subsets of X. we say that A *is included in* B and write $A \subseteq B$ if

$$\mu_A(x) \leq \mu_B(x) \text{ for all } x \in X.$$

Remark. Note that the empty fuzzy set is included in any fuzzy subset A. Also, if $x \in X$ and A is a fuzzy subset of X, then

$$\{x\} \subseteq A \text{ if and only if } \mu_A(x) = 1.$$

Definition II. 2. Let X be a set and R a fuzzy subset of $X \times X$ characterized by $r : X \times X \to [0, 1]$. Then we call R a *fuzzy relation* on X. If a fuzzy relation R satisfies the following properties

(i) for all $x \in X$, $r(x, x) \in [0, 1]$ (reflexivity),

(ii) for all $x, y \in X$, $r(x, y) + r(y, x) > 1$ implies $x = y$ (antisymmetry), and

(iii) for all $x, y, z \in X$, $r(x, y) \geq r(y, x)$ and $r(y, z) \geq r(z, y)$ implies $r(x, z) \geq r(z, x)$ (transitivity),

then we say that R is a *fuzzy order* on X and X is a *fuzzy ordered set.*

Definition II. 3. Let X be a set with fuzzy relation R satisfying (iii). Then R is said to be *f-total* if for all $x \neq y$ we have either $r(x, y) > r(y, x)$ or $r(y, x) > r(x, y)$. A subset of X on which the fuzzy order is *f*-total is called a *f-chain.*

Let A be a subset of X. We say that $x \in X$ is an *f-upper bound* of A if $r(y, x) \geq r(x, y)$ for all $y \in A$. Also an element u is called an *f-supremum* of A and denoted by $u = f\text{-}\sup A$ if x is an upper bound of A and for all upper bound u of A, $r(u, x) \geq r(x, u)$ holds.

If x is an f-upper bound of A and $x \in A$, then x is called an *f-greatest element* of A.

An element $x \in A$ is called an *f-maximal element* of A if there is no $y \neq x$ in A for which $r(x, y) \geq r(y, x)$.

Similary, we can define *f-lower bound*, *f-least element*, *f-minimal element*, and *f*-inf of A.

Let X be a set with a fuzzy relation R satisfying (iii) and define a relation

$\preceq$ on X by

$$x \preceq y \Longleftrightarrow r(x,y) \geq r(y,x)$$

for all $x, y \in X$.

Then it is easy to show that $\preceq$ is a reflexive and transitive relation. That is, $(X, \preceq)$ is a quasi-ordered set. We call $\preceq$ the *quasi-order induced by fuzzy relation* R. Also the followings are true;

I. If $C \subseteq X$ is f-chain, then C is a chain with respect to $\preceq$.

II. For any subset $A \subseteq X$ and $x_0 \in X$, x_0 is an f-upper bound of A if and only if x_0 is a upper bound of A. Also

$$x_0 = f\text{-}\sup A \Longleftrightarrow x_0 = \sup A$$

III. Let $A \subseteq X$ and $x_0 \in X$, if x_0 is a f-maximal element of A, then x_0 is a maximal element of A.

The following Examples will show the relation between the fuzzy maximal elements and the usual maximal elements with respect to $\preceq$.

Example 1. Let X=[0,1] and define $r : X \times X \to [0,1]$ by

$$r(x,y) = \begin{cases} 1, & \text{if } x = y \\ 0, & \text{if } x \neq y \end{cases}$$

Let R be the fuzzy subset of $X \times X$ characterized by r. Then X is a fuzzy ordered set. Indeed, we have;

(i) is obvious.

(ii) $r(x,y) + r(y,x) > 1 \Rightarrow r(x,y) = r(y,x) = 1 \Rightarrow x = y$.

(iii) If $r(x,y) \geq r(y,x)$ and $r(y,z) \geq r(z,y)$, then $r(x,z) \geq r(z,x)$ by the following arguments.

Case 1. If x, y, z are all distinct, then $r(x,z) = 0 = r(z,x)$.

Case 2. If $x = y \neq z$, then

$$r(x,y) = r(y,x) = 1,\ r(y,z) = 0 = r(z,y),\ r(x,z) = 0 = r(z,x).$$

Case 3. If $x \neq y = z$, then $r(x,z) = 0 = r(z,x)$.

Case 4. If $x = z \neq y$, then $r(x,z) = 1 = r(z,x)$.

Case 5. If $x = y = z$, then $r(x,z) = 1 = r(z,x)$.

Hence r is a fuzzy order on X.

Let $x \in X$ be any element. If $y \in X$ and $x \neq y$, then $r(x,y) = r(y,x) = 0$. And if $x = y$, then $r(x,y) = r(y,x) = 1$. In any case, $x \preceq y$ implies $y \preceq x$. Hence x is maximal with respect to $\preceq$.

This shows that every element in X is maximal. Note that X does not have any f-maximal element.

Let C be a nonempty f-chain in X. If $x \neq y$ are in C, then $r(x,y) = r(y,x) = 0$, which contradicts the f-totality of C. Hence C must be a singleton. Therefore, every chain in X has a supremum.

Remark. Let X be any set with fuzzy relation R satisfying (iii). If $r(x, y) = r(y, x)$ for all $x \neq y$ in X, then X does not have any f-maximal element as in Example 1.

Example 2. Let X=[0,1] and define $r : X \times X \to [0, 1]$ by

$$r(x,y) = \begin{cases} 1, & \text{if } x = y \\ \frac{1}{2}, & \text{if } x < y \\ 0, & \text{if } x > y \end{cases}$$

Let R be the fuzzy subset of $X \times X$ characterized by r. Then X is a fuzzy ordered set. Indeed, we have;

(i) $r(x, x) = 1$

(ii) $r(x, y) + r(y, x) > 1 \Rightarrow r(x, y) = r(y, x) = 1 \Rightarrow x = y$

(iii) If $r(x, y) \geq r(y, x)$ and $r(y, z) \geq r(z, y)$, then $r(x, z) \geq r(z, x)$ by the following arguments.

Case 1. $x < y < z \Rightarrow \frac{1}{2} = r(x, y) \geq r(y, x) = 0,\ \frac{1}{2} = r(y, z) \geq r(z, y) = 0$

$\Rightarrow \frac{1}{2} = r(x, z) \geq r(z, x) = 0$

Case 2. $x = y < z \Rightarrow 1 = r(x, y) \geq r(y, x) = 1,\ \frac{1}{2} = r(y, z) \geq r(z, y) = 0$

$\Rightarrow \frac{1}{2} = r(x, z) \geq r(z, x) = 0$

Case 3. $x < y = z \Rightarrow \frac{1}{2} = r(x, y) \geq r(y, x) = 0,\ 1 = r(y, z) \geq r(z, y) = 1$

$\Rightarrow \frac{1}{2} = r(x, z) \geq r(z, x) = 0$

Case 4. $x = y = z \Rightarrow 1 = r(x, y) \geq r(y, x) = 1,\ 1 = r(y, z) \geq r(z, y) = 1$

$\Rightarrow 1 = r(x, z) \geq r(z, x) = 1$

Note that the following is true;

$$r(x, y) = r(y, x) \Longleftrightarrow x = y.$$

Hence every element of X is a maximal element with respect to $\preceq$.

We will show that 1 is the only f-maximal element.

For any $x \neq 1$, there exists some y in X such that $y > x$. Then $r(x, y) = \frac{1}{2} > r(y, x) = 0$. Hence x is not an f-maximal element.

If $x = 1$, then for all $y \in X$=[0, 1], $y \lneqq x$ implies that $r(x, y) = 0 < r(y, x) = \frac{1}{2}$.

Hence 1 is the only f-maximal element.

Let X be a nonempty set with a fuzzy relation R on $X \times X$. We consider the following additional condition;

(iv) $r(x, y) = r(y, x)$ implies $x = y$.

Note that if X is an f-chain, then (iv) holds.

Definition II. 4. A map $f : X \to X$ is said to be *fuzzy monotone* if $r(x, y) \leq r(y, x)$ implies that $r(f(x), f(y)) \leq r(f(y), f(x))$. Hence f is *f-monotone* if and only if f is $\preceq$*-increasing.*

Huy [5] and Huy and Khanh [6] proved some fixed point theorems for multivalued increasing operators on ordered Banach spaces. They defined a multivalued operator $T : X \to 2^X \setminus \{\emptyset\}$ to be increasing if $x \preceq y$ implies that for all $z \in T(x)$, there exists a $w \in T(y)$ such that $z \preceq w$.

Let X be a fuzzy ordered set. A fuzzy multivalued map is any map $T : X \to [0,1]^X \setminus \{\emptyset\}$ such that for every $x \in X$, $T(x)$ is a nonempty fuzzy subset of X. The fuzzy multivalued map $T : X \to [0,1]^X \setminus \{\emptyset\}$ is said to be *fuzzy monotone* if and only if for every $x, y \in X$ such that $x \preceq y$ implies that for all $\{a\} \subseteq T(x)$ there exists $\{b\} \subseteq T(y)$ such that $a \preceq b$.

A point $x \in X$ is called a *fixed point* of a fuzzy multivalued map $T : X \to [0,1]^X \setminus \{\emptyset\}$ is $\{x\} \subseteq T(x)$.

A multivalued map $T : X \to 2^X \setminus \{\emptyset\}$ can be considered as a fuzzy multivalued map $T : X \to [0,1]^X \setminus \{\emptyset\}$ defined by

$$T_{\{x\}}(y) = \begin{cases} 1, & \text{if } y \in T(x) \\ 0, & \text{if } y \notin T(x) \end{cases}$$

for all $x, y \in X$.

III. Main results

In this section, we will prove fuzzy Zorn's lemma using usual Zorn's lemma in quasi-ordered sets. First, we begin with the following;

Theorem 1 (Hausdorff Maximality Principle). *Every fuzzy ordered set X has a maximal f-chain under inclusion.*

Proof. Let Σ be the set of all f-chains in X. Order Σ by set inclusion.

For any chain $\{C_\alpha\}_{\alpha\in\Lambda}$ in Σ, let $D = \bigcup_\alpha C_\alpha$.

If $x \neq y \in D$, then $x \in C_\alpha$ and $y \in C_\beta$ for some $\alpha, \beta \in \Lambda$. Since $C_\alpha \subseteq C_\beta$ or $C_\beta \subseteq C_\alpha$, x and y are in a same set, say C_β. Then $r(x,y) > r(y,x)$ or $r(y,x) > r(x,y)$. Hence D is an f-chain.

Hence $D \in \Sigma$ and is the supremum of $\{C_\alpha\}_{\alpha\in\Lambda}$.

Suppose that Σ does not have a maximal f-chain. Then for all $C \in \Sigma$, there is a f-chain D_C such that $C \subsetneqq D_C$.

If we define $T : \Sigma \to \Sigma$ by $T(C) = D_C$, then for all $C \in \Sigma$,

$$C \subseteq T(C).$$

Hence by Zermelo's fixed point theorem, T has a fixed point, which contradicts the assumption. Therefore, Σ has a maximal element.

The following is due to Beg [1]. But we will prove it using Zorn's lemma in general ordered set.

Theorem 2 (Fuzzy Zorn's Lemma). *Let X be a fuzzy ordered set on which (iv) holds. If every f-chain in X has an f-upper bound, then X has an f-maximal element.*

Proof. As in the proof of Theorem 1, let Σ be the set of all f-chains in X. By theorem 1, Σ has a maximal element, say C_0. Let u be an upper bound of C_0. We will show that u is an f-maximal element of X.

Assume that u is not an f-maximal element. Then there exists $y \neq u$ in X such that $r(u,y) \geq r(y,u)$.

For any $x \in C_0$,

$$r(x,u) \geq r(u,x) \text{ and } r(u,y) \geq r(y,u)$$

implies that $r(x,y) \geq r(y,x)$. Hence y is an upper bound of C_0.

If $y \notin C_0$, then $C_0 \cup \{y\}$ is an f-chain and $C_0 \subsetneqq C_0 \cup \{y\}$, which contradicts the maximality of C_0.

Hence $y \in C_0$ and $r(y,u) \geq r(u,y)$, since u is an upper bound of C_0, but then $r(y,u) = r(u,y)$.

Since (iv) holds, then $u = y$, which contradicts the assumption that $y \neq u$.

Therefore, u is a maximal element.

Remark. If (iv) does not hold, then as in the Example 1 in the previous section, Theorem 2 does not hold.

The following fixed point theorem for fuzzy multivalued maps is given by Stouti [10].

Theorem 3. *Let X be a fuzzy ordered set with the property that (iv) holds and every f-chain in X has a supremum. Let $T : X \to [0,1]^X \setminus \{\emptyset\}$ be a fuzzy monotone multifunction such that* $\sup T(x)$ *exists and* $\{\sup T(x)\} \subseteq T(x)$, *for all $x \in X$. If there exist $a, b \in X$ such that $\{b\} \subseteq T(a)$ and $r(a,b) \geq r(b,a)$, then the set of fixed points of T is nonempty and has a maximal element.*

Proof. Let Σ be the set of all f-chains in C such that for all $x \in C$, there exists $y \in X$ satisfying $\{y\} \subseteq T(x)$ and

$$r(x,y) \geq r(y,x).$$

Then $\{a\} \in \Sigma$ and hence $\Sigma \neq \emptyset$. Order Σ by set inclusion.

Assume that Σ does not have a maximal element. Then for any $C \in \Sigma$, there is $D_C \in \Sigma$ such that $C \subsetneq D_C$. Define $F : \Sigma \to \Sigma$ by $F(C) = D_C$.

For any chain $\{C_\alpha\}_{\alpha \in \Lambda}$ in Σ, let $D = \bigcup C_\alpha$.

If $x \neq y \in D$, then $x \in C_\alpha$ and $y \in C_\beta$ for some $\alpha, \beta \in \Lambda$. Since $C_\alpha \subseteq C_\beta$ or $C_\beta \subseteq C_\alpha$, x and y are in a same set, say C_β. Then $r(x,y) > r(y,x)$ or $r(y,x) > r(x,y)$. Hence D is an f-chain.

For all $x \in D$, since it is a member of some C_α, there exists $y \in X$ such that $\{y\} \subseteq T(X)$ and $r(x,y) \geq r(y,x)$. Hence $D \in \Sigma$ and hence D is the supremum of $\{C_\alpha\}_{\alpha\in\Lambda}$.

By Zermelo's fixed point theorem, F has a fixed point, which contradicts the fact that $C \neq D_C$. So Σ has a maximal element, say C_0.

Since C_0 is an f-chain, it has a least upper bound $u \in X$.

For any $x \in C_0$, choose $y \in X$ such that $\{y\} \subseteq T(x)$ and $r(x,y) \geq r(y,x)$. Then $r(x,u) \geq r(u,x)$ implies that there is $v \in X$ such that $\{v\} \subseteq T(u)$ and

$$r(y,v) \geq r(v,y),$$

since T is f-monotone.

But then $x \preceq y$ and $y \preceq v$ implies that $x \preceq v$. Hence v is an upper bound for C_0.

Since u is the least upper bound for C_0, we have

$$r(u,v) \geq r(v,u).$$

But then $C_0 \cup \{u\} \in \Sigma$. Since C_0 is maximal, $u \in C_0$ and u is the greatest element of C_0. Similarly, $v \in C_0$, and hence $u = v$ and

$$\{u\} = \{v\} \subseteq T(u).$$

Therefore T has a fixed point u. It is obvious that u is a maximal fixed point.

Now we obtain a fuzzy version of the Huy and Khanh's theorem [6].

Theorem 4. *Let X be a nonempty set and R be a fuzzy order on X. Assume that (iv) holds and every f-chain in X has a supremum. Let $F : X \to 2^X \setminus \{\emptyset\}$ be a monotone multivalued function with respect to $\preceq$ defined as above. Suppose that $\sup F(x)$ exists and $\sup F(x) \in F(x)$ for all $x \in X$.*

If there exist $a, b \in X$ such that $b \in F(a)$ and $a \preceq b$, then F has a maximal fixed point.

Proof. Let $T : X \to [0,1]^X \setminus \{\emptyset\}$ be defined by

$$T_{\{x\}}(y) = \begin{cases} 1, & \text{if } y \in F(x) \\ 0, & \text{otherwise.} \end{cases}$$

Then T is a fuzzy multivalued map.

By assumption, there exist $a, b \in X$ such that $b \in F(a)$ and $a \preceq b$. Hence

$$T_{\{a\}}(b) = 1 \text{ and so } \{b\} \subseteq T(a).$$

Assume that $x \preceq y$ and $\{a\} \subseteq T(x)$, then $T_{\{x\}}(a) = 1$ implies that $a \in F(x)$. Since F is monotone, there exists $b \in F(y)$ such that $a \preceq b$. Therefore

$$r(a, b) \geq r(b, a).$$

Since $b \in F(y)$, we have $\{b\} \subseteq T(y)$. Hence T is f-monotone. By theorem 3, T has a maximal fixed point u. Since $\{u\} \subseteq T(u)$, $T_{\{u\}}(u) = 1$, that is, $u \in F(u)$. This completes the proof.

Note that Huy and Khanh proved some fixed point theorem on ordered

Banach space with strongly miniheadral cone.

The following is the single valued version of Theorem 4, which is due to Beg [2].

Corollary. Let X be a nonempty set and R be a fuzzy order on X. Assume that (iv) holds and every f-chain in X has a supremum. Let $f : X \to X$ be an f-monotone map such that $r(a, f(a)) \geq r(f(a), a)$ for some $a \in A$. Then f has a maximal fixed point.

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ABSTRACT

It is well-known that maximal principles on general ordered sets can be reformulated by fixed point theorems.

Beg [2] showed that fuzzy Zorn's lemma holds on fuzzy ordered sets. Related fixed point theorems for single valued and multivalued fuzzy maps have been studied by Beg [1, 2, 3] and Stouti [10].

In this paper, we show that fuzzy Zorn's lemma can be derived from usual Zorn's lemma. To do this, we show that Hausdorff maximality principle holds in fuzzy ordered sets.

We prove some fixed point theorems for single valued and multivalued fuzzy maps. We obtain a fuzzy version of Huy and Khanh's fixed point theorem for multivalued operators in ordered Banach space[6]. It is a generalization of the fixed point theorem of Beg [2] for single valued fuzzy monotone map.