沈聖娥 教授指導

碩士學位 請求論文

Mathematical modeling of various diffusion mechanisms

확산 메커니즘의 수학적 모델링

2008年

誠信女子大學校 教育大學院

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韓丞希

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논문개요

세포, 박테리아, 화학약품, 동물 등과 같은 입자들의 집합에서 각각의 입자들은 일반적으로 임의의 방향으로 움직인다. 입자들은 이 불규칙한 개별적인 입자의 움직임의 결과로 퍼져나간다. 이 미세한 불규칙적 움직임이 그 집합의 거시적 혹은 총체적인 규칙적 움직임으로 끝날때 우리는 이것을 확산과정이라고 생각할 수 있다. 이러한 입자들은 그 불규칙적인 입자들의 움직임이 총체적으로 간단한 확산으로 끝나지 않는 경우에 오차가 발생하는 환경과 상호작용이 있다. 하지만 입자들의 개별적이고 미세한 행동의 움직임에서 거시적인 행동을 얻는것은 매우 힘들다. 그래서 이 논문은 입자의 밀도나 농도를 이용해서 전체적인 행동을 위한 연속체 모형 방정식을 유도한다. 또한 여러가지 입자 모형을 통해 연속체 모형 방정식을 확인하는 것을 목적으로 한다. 그것은 우리가 초보적인 방법으로 입자의 움직임에 관한 모형 방정식 유도를 시작하는데 도움을 주고, 그 후에 결정적인 모형을 유도할 수 있게 한다.

차 례

I Introduction

In an assemblage of particles, for example, cells, bacteria, chemicals, animals and so on,each particle usually moves around in random way. The particles spread out as a result of this irregular individual particle's motion. when this microscopic irregular movement result in some macroscopic or gross regular motion of the group we can think of it as a diffusion process. Of course there may be interaction between particles and the environment mat give some bias in which case the gross movement is not simple diffusion. To get the macroscopic behaviour from a knowledge of the individual microscopic behaviour is much too hard so we drive a continuum model equation for the global behaviour in terms of a particle density or concentration. It is instructive to start with a random process which we look at probabilistically in an elementary way, and then drive a deterministic model. For simplicity we consider initially only one-dimensional motion and the simplest random walk process. the generalisation to higher dimensions is then intuitively clear from the one-dimensional equation. Diffusion models form a reasonable basis for studying insect and animal dispersal and invasion;this and other aspects of animal population models are discussed in detail, for example, by Okubo(1980, 1986), Shigesada(1980) and Lewis(1997). Dispersal of interacting species is discussed by Shigesada et al.(1979) and of competing species by Shigesada and Roughgarden(1982). Kareiva(1983) has shown that many species appear to disperse according to a reaction diffusion model with a constant diffusion coefficient. He gives actual values for the diffusion coefficient which he obtained from experiments on variety of insect species. Kot et al.(1996) studied dispersal of organisms in general and importantly incorporated real data(see also Kot 2001). A common feature of insect populations is their discrete time population growth. As would be expected intuitively this can have a major effect on their spatial dispersal. The model equations involve the coupling of discrete time with continuous space, a topic investigated by Kot(1992) and Neubert et al.(1995).

II Preliminaries

Suppose a particle moves randomly backward and forward along a line in fixed steps Δx that are taken in a fixed time Δt. If the motion is unbiased then it is equally probable that the particle takes a step to the right or left. After time $N\Delta t$ the particle can be anywhere from $-N\Delta x$ to $N\Delta x$ if we take the starting point of the particle as the origin. The spatial distribution is clearly not going to be uniform if we release a group of particles about $x = 0$ since the probability of a particle reaching $x = N\Delta x$ after N steps is very small compared with that for x nearer $x = 0$.

Lemma 1. *The probability $p(m, n)$ that a particle reaches a point m space steps to the right can be approximated for large m, n as*

$$p(m,n) \sim \left(\frac{2}{\pi n}\right)^{1/2} e^{\frac{-m^2}{(2n)}}. \tag{1}$$

Proof. Let us suppose that to reach $m\Delta x$ it has moved a steps to the right and b to the left. Then

$$m = a - b, \quad a + b = n,$$

and so

$$a = \frac{n+m}{2}, \quad b = n - a.$$

The number of possible paths that a particle can reach this point $x = m\Delta x$ is

$$\frac{n!}{a!b!} = \frac{n!}{a!(n-a)!} C_a^n,$$

where C_a^n is the binomial coefficient. The total number of possible n-step paths is 2^n and so the probability $p(m,n)$ is

$$p(m,n) = \frac{1}{2^n} \frac{n!}{a!(n-a)!}, \tag{2}$$

with $a = \frac{n+m}{2}$ when $m+n$ is even.

Note that

$$\sum_{m=-n}^{n} p(m,n) = \sum_{a=0}^{n} C_a^n \left(\frac{1}{2}\right)^{n-a} \left(\frac{1}{2}\right)^{a} = \left(\frac{1}{2} + \frac{1}{2}\right)^{n} = 1,$$

and thus $p(m,n)$ is the *binomial distribution.*

If we now let n be large so that $n \pm m$ are also large we have, asymptotically,

$$n! \sim (2\pi n)^{\frac{1}{2}} n^n e^{-n}, n \gg 1, \tag{3}$$

which is Stirling's formula. Using (2) in (1) we get the *normal* or *Gaussian probability distribution*

$$p(m,n) \sim \left(\frac{2}{\pi n}\right)^{1/2} e^{\frac{-m^2}{(2n)}}, \quad m \gg 1, \quad n \gg 1.$$

□

For (1) m and n need not be very large to be an accurate approximation to (2). In fact for all practical purposes we can use (1) for $n > 6$. Asymptotic approximations can often be remarkably accurate over a wider range than might be imagined.

Lemma 2. *Suppose a particle moves randomly along the $x-$axis with the condition that*

$$\lim_{\Delta x \to 0_{\Delta t \to 0}} \frac{(\Delta x)^2}{2\Delta t} = D \neq 0.$$

Then the probability $u(x,t)$ finding the particle at the location x at time t is

$$u(x,t) = \lim_{\Delta x \to 0_{\Delta t \to 0}} \frac{p(\frac{x}{\Delta x}, \frac{t}{\Delta t})}{2\Delta x} = \left(\frac{1}{4\pi D t}\right)^{\frac{1}{2}} e^{-x^2/(4Dt)}. \tag{4}$$

Proof. Let us set

$$m\Delta x = x, \quad n\Delta t = t,$$

where x and t are the continuous space and time variables. If we anticipate letting $m \to \infty, n \to \infty, \Delta x \to 0, \Delta t \to 0$ so that x and t are finite , then it is not appropriate to have $p(m,n)$ as the quantity of interest since this probability must tend to zero the number of points on the line tends to ∞ as $\Delta x \to 0$. The relevant dependent variable is more appropriately $u = \frac{p}{2\Delta x}$, where $2u\Delta x$ is the probability of finding a particle in the interval $(x, x+\Delta x)$ at time t. From (1) with $m = \frac{x}{\Delta x}$, $n = \frac{t}{\Delta t}$,

$$\frac{p(\frac{x}{\Delta x}, \frac{t}{\Delta t})}{2\Delta x} \sim \left\{\frac{\Delta t}{2\pi t (\Delta x)^2}\right\}^{\frac{1}{2}} \exp\left\{-\frac{x^2}{2t}\frac{\Delta t}{(\Delta x)^2}\right\}$$

If we assume

$$\lim_{\Delta x \to 0_{\Delta t \to 0}} \frac{(\Delta x)^2}{2\Delta t} = D \neq 0$$

the last equation gives

$$u(x,t) = \lim_{\Delta x \to 0_{\Delta t \to 0}} \frac{p(\frac{x}{\Delta x}, \frac{t}{\Delta t})}{2\Delta x} = \left(\frac{1}{4\pi D t}\right)^{\frac{1}{2}} e^{-x^2/(4Dt)}.$$

□

The quantity D represents the *diffusion coefficient* or *diffusivity* of the particles.

Let us now relate this result to the classical approach to diffusion, namely, *Fickian diffusion.* This says that the flux, J, of material, which can be cells, amount of chemical, number of animals and so on, is proportional to the gradient of the concentration of the material. That is, in one dimension

$$J \propto -\frac{\partial c}{\partial x} \quad \Rightarrow \quad J = -D\frac{\partial c}{\partial x}, \tag{5}$$

where $c(x,t)$ is the concentration of the species and D is its diffusivity. The minus sign simply indicates that diffusion transports matter from a high to a low concentration. We now write a general conservation equation which says that the rate of change of the amount of material in a region is equal to the rate of flow across the boundary. If the region is $x_0 < x < x_1$ and no material is created,

$$\frac{\partial}{\partial t}\int_{x_0}^{x_1} c(x,t)dx = J(x_0,t) - J(x_1,t). \tag{6}$$

If we take $X_1 = x_0 + \Delta x$, take the limit as $\Delta x \to 0$ and use (5) we get the classical diffusion equation (7) in one dimension,

$$\lim_{\Delta x\to 0}\frac{\frac{\partial}{\partial t}\int_{x_0}^{x_0+\Delta x} c(x,t)dx}{\Delta x} = \lim_{\Delta x\to 0}\frac{J(x_0,t) - J(x_0+\Delta x,t)}{\Delta x}$$

$$\frac{\partial}{\partial t}c = -\frac{\partial J}{\partial x}$$

From (5) we have that

$$\frac{\partial c}{\partial t} = -\frac{\partial J}{\partial x} = \frac{\partial (D\frac{\partial c}{\partial x})}{\partial x} \tag{7}$$

which, if D is constant, becomes

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2} \tag{8}$$

III Main Results

Theorem 3. *If we release an amount Q of particles per unit area at $x = 0$ at $t = 0$, that is,*

$$c(x, 0) = Q\delta(x) \tag{9}$$

where $\delta(x)$ is Dirac delta function, then the solution of (8) *is*

$$c(x, t) = \frac{Q}{2(\pi Dt)^{\frac{1}{2}}} e^{-\frac{x^2}{4Dt}}, t > 0 \tag{10}$$

which, with $Q = 1$, is the same result as (4), *obtained from a random walk approach when the step and time size are small compared with x and t.*

Proof. We apply the Fourier Transform

$$\mathfrak{F}[f] = F(\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\xi x} dx$$

on both sides of the equation (8) to obtain that

$$\mathfrak{F}\left[\frac{\partial c}{\partial t}\right] = \mathfrak{F}\left[D\frac{\partial^2 c}{\partial x^2}\right].$$

Simplifying it gives

$$\begin{aligned} \frac{\partial}{\partial t}\mathfrak{F}[c] &= D\mathfrak{F}[\frac{\partial^2 c}{\partial x^2}] \\ &= D(-\xi)^2\mathfrak{F}[c] \\ &= -D\xi^2\mathfrak{F}[c]. \end{aligned}$$

We solve the differential equation of the type

$$\frac{\partial y}{\partial t} = y' = -D\xi^2 y, \quad \text{equivalently,} \quad \frac{\partial y}{y} = -D\xi^2 \partial t$$

with the initial condition

$$\begin{aligned} \mathfrak{F}[c(x,0)] &= \mathfrak{F}[Q,\delta(x)] \\ &= Q\mathfrak{F}[\delta(\mathfrak{x})] \end{aligned}$$

in the form

$$y(0) = \frac{Q}{\sqrt{2\pi}}e^0.$$

Thus the differential equation becomes

$$\frac{\partial y}{\partial t} = y' = -D\xi^2 y$$

$$\frac{\partial y}{y} = -D\xi^2 \partial t.$$

The solution of the initial value problem is

$$\ln|y| = -D\xi^2 t + \widetilde{c},$$

$$y = ce^{-D\xi^2 t}.$$

Since $y(0) = c$, we have the solution

$$y(t) = y(0)e^{-D\xi^2 t},$$

and thus

$$\begin{array}{rcl} \mathfrak{F}[c] & = & y(t) \\ & = & y(0)e^{-D\xi^2 t} \\ & = & \frac{Q}{\sqrt{2\pi}}e^{-D\xi^2 t} \end{array}$$

Now by applying the inverse Fourier Transform we obtain that

$$\begin{array}{rcl} c(x,t) & = & \mathfrak{F}^{-1}[\frac{Q}{\sqrt{2\pi}}e^{-D\xi^2 t}] \\ & = & \frac{Q}{\sqrt{2\pi}}\mathfrak{F}^{-1}[e^{-D\xi^2 t}] \\ & = & \frac{Q}{\sqrt{2\pi}}e^{-D\xi^2 t}(a = \frac{1}{\sqrt{4Dt}}) \\ & = & \frac{Q}{\sqrt{2\pi}}\mathfrak{F}^{-1}[e^{\frac{-\xi^2}{4a^2}}] \\ & = & \frac{Q}{\sqrt{2\pi}}a\sqrt{2}e^{-a^2x^2} \\ & = & \frac{Q}{\sqrt{\pi}}\frac{1}{\sqrt{4Dt}}e^{-\frac{x^2}{4Dt}} \\ & = & \frac{Q}{2(\pi Dt)^{\frac{1}{2}}}e^{-\frac{x^2}{4Dt}}, t > 0 \end{array}$$

$$c(x,t) = \frac{Q}{2(\pi D t)^{\frac{1}{2}}} e^{-\frac{x^2}{4Dt}}, t > 0 \tag{11}$$

which, with $Q = 1$, is the small result as (4), obtained from a random walk approach when the step and time size are small compared with x and t. $\square$

Now we drive a way of relating the diffusion equation to the random walk approach using $p(x,t)$, from (4), as the probability that a particle released at $x = 0$ at $t = 0$ reaches x in time t. At time $t - \Delta t$ the particle was at $x - \Delta x$ or $x + \Delta x$. Thus, if α and β are probabilities that a particle will move to the right or left

$$p(x,t) = \alpha p(x - \Delta x, t - \Delta t) + \beta p(x + \Delta x, t - \Delta t), \quad \alpha + \beta = 1 \tag{12}$$

If there is no bias in the random walk, that is, it is isotropic, $\alpha = \frac{1}{2} = \beta$.

Theorem 4. *The probability function* $p(x,t)$ *satisfies that*

$$\frac{\partial p}{\partial t} = D \frac{\partial^2 p}{\partial x^2}.$$

If the total number of released particles is Q*, then the concentration of particles* $c(x,t) = Qp(x,t)$ *and the last equation becomes* (8).

Proof. Expanding the right-hand side of (12) in a Taylor series,

$$\begin{aligned}
p(x,t) &= \frac{1}{2}p(x-\Delta x, t-\Delta t) + \frac{1}{2}p(x+\Delta x, t-\Delta t) \\
&= \{\frac{1}{2}p(x,t) + \frac{1}{2}p(x,t)\} + \{\frac{1}{2}\frac{\partial p}{\partial x}(-\Delta x) + \frac{1}{2}\frac{\partial p}{\partial t}(-\Delta t)\} \\
&\quad +\{\frac{1}{2}\frac{\partial p}{\partial x}(\Delta x) + \frac{1}{2}\frac{\partial p}{\partial t}(-\Delta t)\} \\
&\quad +\frac{1}{2}\frac{1}{2!}\{\frac{\partial^2 p}{\partial x^2}(\Delta x)^2 + 2(-\Delta x)(-\Delta t)\frac{\partial^2 p}{\partial x \partial t} + (\Delta t)^2\frac{\partial^2 p}{\partial t^2}\} + \cdots
\end{aligned}$$

$$p(x,t) = p(x,t) - \frac{\partial p}{\partial t}(\Delta t) + \frac{1}{2}\frac{\partial^2 p}{\partial x^2}(\Delta x)^2 + \frac{1}{2}\frac{\partial^2 p}{\partial t^2}(\Delta t)^2 + \cdots$$

$$\frac{\partial p}{\partial t} = [\frac{(\Delta x)^2}{2\Delta t}]\frac{\partial^2 p}{\partial x^2} + (\frac{\Delta t}{2})\frac{\partial^2 p}{\partial t^2} + \cdots$$

Let $\Delta x \to 0, \Delta t \to 0$

$$\lim_{\Delta x \to 0_{\Delta t \to 0}} \frac{(\Delta x)^2}{2\Delta t} = D$$

We get

$$\frac{\partial p}{\partial t} = D\frac{\partial^2 p}{\partial x^2}$$

If the total number of released particles is Q, then the concentration of particles $c(x,t) = Qp(x,t)$ and the last equation becomes (8). □

Reaction Diffusion Equation in three space dimensions

Consider now diffusion in three space dimensions. Let S be an arbitrary surface enclosing a volume V. The general conservation equation says that the rate of change of the amount of material in V is equal to the rate of flow of material across S into V plus the material created in V. Thus

$$\frac{\partial}{\partial t}\int_V c(x,t)dv = -\int_S \mathbf{J}ds + \int_V f dv, \tag{13}$$

where $\mathbf{J}$ is the flux of material and f, which represents the source of material, may be a function of c, $\mathbf{x}$ and t. Applying the divergence theorem to the surface integral and assuming $c(\mathbf{x},t)$ is continuous, the last equation becomes

$$\int_V \left[\frac{\partial c}{\partial t} + \nabla\cdot J - f(c,\mathbf{x},t)\right] dv = 0. \tag{14}$$

Since the volume V is arbitrary the integrand must be zero and so the *conservation equation* for c is

$$\frac{\partial c}{\partial t} + \nabla\cdot\mathbf{J} = f(c,\mathbf{x},t) \tag{15}$$

This equation holds for a general flux transport $\mathbf{J}$, whether by diffusion or some other process.

If classical diffusion is the process then the generalization of (5), for example, is

$$\mathbf{J} = -D\nabla c \tag{16}$$

and (15) becomes

$$\frac{\partial c}{\partial t} = f + \nabla \cdot (D\nabla c) \tag{17}$$

The source term f in an ecological context, for example, could represent the birth-death process and c the population density, n. With logistic population growth $f = rn(1 - \frac{n}{K})$, where r is the linear reproduction rate and K the carrying capacity of the environment. The resulting equation with D constant is

$$\frac{\partial n}{\partial t} = rn(1 - \frac{n}{K}) + D\nabla^2 n \tag{18}$$

known as the *Fisher-Kolmogoroff equation.*

If we further generalize (17) to the situation in which there are, for example, several interacting species or chemicals we then have a vector $u_i(x,t)$, $i = 1, \ldots, m$ of densities or concentrations each diffusing with its own diffusion coefficient D_i and interacting according to the vector source term $\mathbf{f}$.

(17) becomes

$$\frac{\partial \mathbf{u}}{\partial t} = \mathbf{f} + \nabla \cdot (D \nabla \mathbf{u}), \tag{19}$$

where D is a matrix of diffusivities, and $\nabla \mathbf{u}$ is a tensor so $\nabla \cdot D \nabla \mathbf{u}$ is a vector.

Models for Animal Dispersal

Diffusion models form a reasonable basis for studying insect and animal dispersal and invasion. Dispersal of interacting species is discussed by Shigesada et al.(1979) and of competing species by Shigesada and Roughgarden(1982). A common feature of insect populations is their discrete time population growth. As would be expected intuitively this can have a major effect on their spatial dispersal.

One extension of the classical diffusion model which of particular relevance to insect dispersal is when there is an increase in diffusion due to population pressure. One such model has the diffusion coefficient, or rather the flux $\mathbf{J}$, depending on the population density n such that D increases with

n ; that is,

$$\mathbf{J} = -D(n)\nabla n, \qquad \frac{\partial D}{\partial n} > 0. \tag{20}$$

A typical form for $D(n)$ in (20) is $D_0(\frac{n}{n_0})^m$, where $m > 0$ and D_0 and n_0 are positive constants.

Theorem 5. *If* $D(n) = D_0(\frac{n}{n_0})^m$ *in* (20) *with* $m > 0$ *and positive constants* D_0 *and* n_0*, then the dispersal equation for* n *without any growth term*

$$\frac{\partial n}{\partial t} = -\nabla \cdot J$$

has an exact analytical solution of the form

$$n(x,t) = \begin{cases} \frac{n_0}{\lambda(t)}[1 - (\frac{x}{r_0}\lambda(t))^2]^{\frac{1}{m}}, & |x| \le r_0\lambda(t) \\ 0, & |x| > r_0\lambda(t) \end{cases} \tag{21}$$

where $\lambda(t) = (\frac{t}{t_0})^{\frac{1}{2+m}}$ *and* r_0, t_0 *are some positive constants.*

Proof. The dispersal equation for n without any growth term becomes

$$\frac{\partial n}{\partial t} = -\nabla \cdot J$$

$$\begin{aligned} \frac{\partial n}{\partial t} &= -\nabla \cdot J \\ &= -\nabla \cdot -D(n)\nabla n \\ &= D_0\nabla \cdot [(\tfrac{n}{n_0})^m \nabla n]. \end{aligned}$$

In one dimension

$$\nabla n = \frac{\partial n}{\partial x}, \nabla \cdot f = \frac{f}{\partial x}$$

$$\begin{aligned} \frac{\partial n}{\partial t} &= D_0 \nabla \cdot [(\tfrac{n}{n_0})^m \nabla n] \\ &= D_0 \nabla \cdot [(\tfrac{n}{n_0})^m \tfrac{\partial n}{\partial x}] \end{aligned}$$

$$\frac{\partial n}{\partial t} = D_0 \frac{\partial}{\partial x} [(\frac{n}{n_0})^m \frac{\partial n}{\partial x}] \tag{22}$$

which has an exact analytical solution of the form

$$n(x,t) = \begin{cases} \frac{n_0}{\lambda(t)} [1 - (\frac{x}{r_0} \lambda(t))^2]^{\frac{1}{m}}, & |x| \leq r_0 \lambda(t) \\ 0, & |x| > r_0 \lambda(t) \end{cases}$$

where

$$\begin{aligned} &\lambda(t) = (\tfrac{t}{t_0})^{\frac{1}{2+m}}, \quad \text{and} \\ &r_0 = \tfrac{Q\Gamma(\frac{1}{m}+\frac{3}{2})}{\pi^{\frac{1}{2}} n_0 \Gamma(\frac{1}{m}+1)}, \qquad t_0 = \tfrac{r_0^2 m}{2D_0(m+2)} \end{aligned} \tag{23}$$

where Γ is the gamma function and Q is the initial number of insects released at the origin. It is straightforward to check that (21) is a solution of (22) for all r_0. □

The evaluation of r_0 comes from requiring the integral of n over all x

to be equal to Q. The population is identically zero for $x > r_0\lambda(t)$. This solution is fundamentallay different from that when $m = 0$, namely, (10). The difference is due to the fact that $D(0) = 0$. The solution represents a kind of wave with the front at $x = x_f = r_0\lambda(t)$. The derivative of n is discontinuous here. The wave 'front' which we define here as the point where $n = 0$, propagates with a speed $\frac{dx_f}{dt} = \frac{r_0 d\lambda}{dt}$, which, from (23), decreases with time for all m. The solution for n is illustrated schematically in figure 2. The dispersal patterns for grasshoppers exhibit a similar behavior to this model. Without any source term the population n, from (21), tends to zero as $t \to \infty$. Shigesada proposed such a model for animal dispersal in which she took the linear diffusion dependence $D(n) \propto n$.

Chemotaxis

Let us suppose that the presence of a gradient in an attractant, $a(x,t)$, gives rise to a movement, of the cells say, up the gradient. The flux of cells will increase with the number if cells, $n(x,t)$, present. Thus we may reasonably take as the chemotactic flux

$$J = n\chi(a)\nabla a, \tag{24}$$

Where $\chi(a)$ is a function of the attractant concentration. In the general conservation equation for $n(x,t)$, namely,

$$\frac{\partial n}{\partial t} + \nabla J = f(n),$$

where $f(n)$ represents the growth term for the cells, the flux

$$\mathbf{J} = \mathbf{J}_{diffusion} + \mathbf{J}_{chemotaxis},$$

where the diffusion contribution is from (15) with the chemotaxis flux from (23). Thus a basic $reaction - diffusion - chemotaxisequation$ is

$$\frac{\partial n}{\partial t} = f(n) - \nabla \cdot n\chi(a)\nabla a + \nabla \cdot D\nabla n, \tag{25}$$

where D is the diffusion coefficient of the cells. Since the attractant $a(x,t)$ is a chemical it also diffuses and is produced, by the amoebae, for example, so we need a further equation for $a(x,t)$. Typically

$$\frac{\partial a}{\partial t} = g(a,t) + \nabla \cdot D_a \nabla_a, \tag{26}$$

where D_a is the diffusion coefficient of a and $g(a,n)$ is the kinetics/source term, which may depend on n and a . Normally we would $D_a > D$. Here hn represents the spontaneous production of the attractant and is proportional to the number of amoebae n , while $-ka$ represents decay of attractant activity; that is, there is an exponential decay if the attractant is not produced by the cells. One simple version of the model has $f(n) = 0$; that is, the amoebae production rate is negligible. This is the case during the pattern formation phase in the mould's life cycle. The chemotactic term $\chi(a)$ is taken to be a positive constant χ_0 . The form of this term has to be determined from experiment. With constant diffusion coefficients, together with the above linear form for $g(a,n)$, the model in one space dimension becomes the nonlinear system

$$
\begin{aligned}
\frac{\partial n}{\partial t} &= f(n) - \nabla \cdot n\chi(a)\nabla a + \nabla \cdot D\nabla n, \\
&= D\frac{\partial}{\partial x}\frac{\partial n}{\partial x} - \frac{\partial}{\partial x}n\chi_0\frac{\partial a}{\partial x} \\
\therefore \frac{\partial n}{\partial t} &= D\frac{\partial^2 n}{\partial x^2} - \chi_0\frac{\partial}{\partial x}(n\frac{\partial a}{\partial x}) \qquad (27) \\
\frac{\partial a}{\partial t} &= g(a,n) + \nabla \cdot D_a\nabla a \\
&= hn - ka + D_a\frac{\partial}{\partial x}\frac{\partial a}{\partial x} \\
\therefore \frac{\partial a}{\partial t} &= hn - ka + D_a\frac{\partial^2 a}{\partial x^2} \qquad (28)
\end{aligned}
$$

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ABSTRACT

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In an assemblage of particles, for example, cells, bacteria, chemicals, animals , and so on each particle usually moves around in random way. The particles spread out as a result of this irregular individual particle's motion. when this microscopic irregular movement result in some macroscopic or gross regular motion of the group we can think of it as a diffusion process. To get the macroscopic behaviour from a knowledge of the individual microscopic behaviour is much too hard so we drive a continuum model equation for the global behaviour in terms of a particle density or concentration. It is instructive to start with a random process which we look at probabilistically in an elementary way, and then drive a deterministic model.